

① a) $\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$

ii) $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ 6 & 8 \end{bmatrix} =$

b) $\begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} = -5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

② $y_{n+1} \approx y_n + h f(x_n)$

$x_1 = 0.5$

$y_1 = 1$

$f(x_1) = \sin(2 \times 0.5) = 0.8414...$

$h = 0.1$

$y_2 \approx 1 + 0.1 \times 0.8414$

$= 1.08414...$

$x_2 = 0.6$

$y_2 = 1.0841...$

$f(x_2) = \sin(2 \times 0.6) = 0.9320...$

$y_3 \approx 1.0841 + 0.1 \times 0.932$

$= 1.1773...$

$= 1.18 \text{ (3sf)}$

③ a) $\sum r^2(r-1) = \sum r^3 - \sum r^2$

$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$

$= \frac{1}{12}n(n+1) [3(n)(n+1) - 2(2n+1)]$

$= \frac{1}{12}n(n+1) [3n^2 + 3n - 4n - 2]$

$= \frac{1}{12}n(n+1) [3n^2 - n - 2]$

$= \frac{1}{12}n(n+1)(3n+2)(n-1)$

$= \frac{1}{12}n(n+1)(n-1)(3n+2)$

$= \frac{1}{12}n(n^2-1)(3n+2)$

↑ Diff of 2 squares

$$b) \sum_{r=4}^{11} r^2(r-1) = \sum_{r=1}^{11} r^2(r-1) - \sum_{r=1}^3 r^2(r-1)$$

$$= \frac{1}{2}(11)(11^2-1)(33+2) - \frac{1}{2}(3)(3^2-1)(9+2)$$

$$= 3850 - 22 = 3828$$

4) a) $f(x) = x^3 + 2x$

$$f(2+h) = (2+h)^3 + (2+h)$$

$$= 8 + 4 \cdot 2h + 6h^2 + h^3 + 2 + h$$

$$= h^3 + 6h^2 + 13h + 10$$

$$f(2) = 2^3 + 2 = 10$$

$$\therefore f(2+h) - f(2) = h^3 + 6h^2 + 13h + 10 - 10$$

$$= h^3 + 6h^2 + 13h$$

OR $13h + 6h^2 + h^3$

b) Gradient = $\frac{13h + 6h^2 + h^3}{h} = 13 + 6h + h^2$

As $h \rightarrow 0$, gradient $\rightarrow 13$

5) a) Tan General solution: $\theta = n\pi + \alpha$

key angle: $\tan^{-1}(\sqrt{3}) = \pi/3$

$$\rightarrow 3\alpha = n\pi + \pi/3$$

$$\rightarrow \alpha = n\pi/3 + \pi/9$$

b) Key angle: $\tan^{-1}(-\sqrt{3}) = -\pi/3$

$$\rightarrow 3\alpha - \pi/3 = n\pi - \pi/3$$

$$\rightarrow 3\alpha = n\pi$$

$$\rightarrow \alpha = n\pi/3$$

6) a) i) $\alpha + \beta = 4$, $\alpha\beta = 13$

ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= 4^2 - 2(13) = -10$$

iii) The square of a real number is ≥ 0

\therefore The sum of squares of real numbers is ≥ 0

$$b) \ i) \ (\alpha + i) + (\beta + i) = \alpha + \beta + 2i$$

$$= 4 + 2i$$

$$ii) \ (\alpha + i)(\beta + i) = \alpha\beta + \alpha i + \beta i + i^2$$

$$= \alpha\beta + i(\alpha + \beta) - 1$$

$$= 13 + 4i - 1 = 12 + 4i$$

$$c) \ \text{sum} = 4 + 2i$$

$$\text{Product} = 12 + 4i$$

$$x^2 - \boxed{\text{sum}}x + \boxed{\text{product}} = 0$$

$$\rightarrow x^2 - (4 + 2i)x + (12 + 4i) = 0$$

$$7) \ a) \ i) \ \left[\begin{array}{ccc|ccc} 1 & & & 3 & 1 & 3 \\ & 1 & & 1 & 1 & 2 \end{array} \right]$$

$$D = (4, 0)$$

$$\left[\begin{array}{cc|cc} 2 & -2 & & \\ -2 & 2 & & \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 4 & & & 8 & 1 & 10 \\ & 0 & & -4 & 1 & -2 \end{array} \right]$$

$$E = (8, -4)$$

$$F = (10, -2)$$

ii) See sheet

$$b) \ i) \ \left[\begin{array}{cc} 2 & 2 \\ -2 & 2 \end{array} \right]$$

From diagram:

$$CB = 1$$

$$FE = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\therefore \text{Enlargement } SF = 2\sqrt{2}$$

$$ii) \ 2\sqrt{2} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(45^\circ) & \sin(45^\circ) \\ -\sin(45^\circ) & \cos(45^\circ) \end{bmatrix}$$

= Rotation 45° clockwise

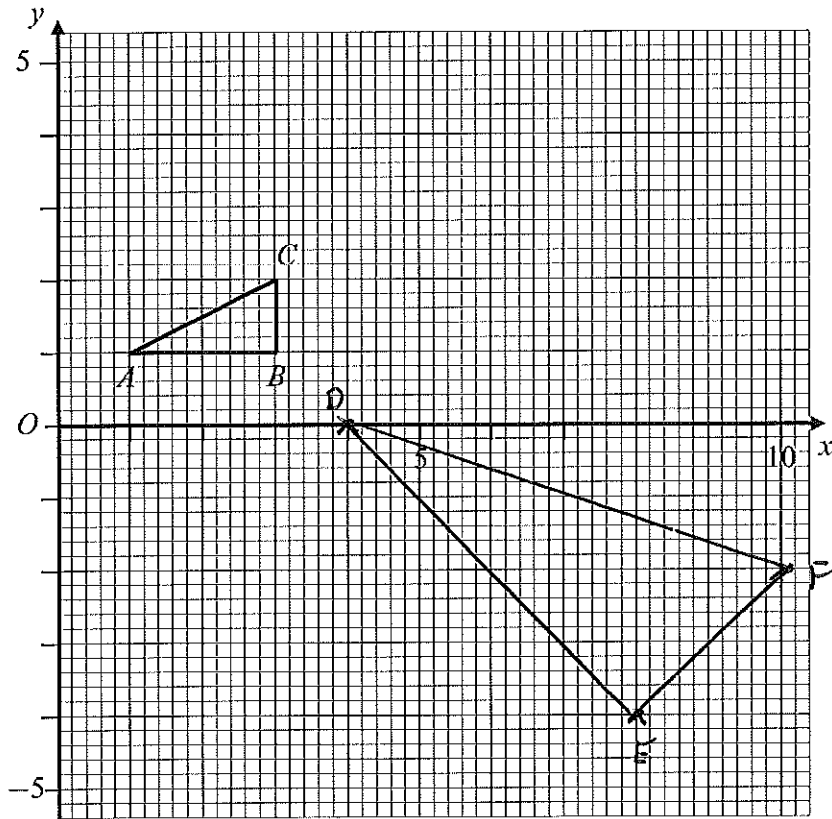


Figure 1 (for use in Question 7)

8) a) At P, $y=0 \rightarrow x=2 \rightarrow (2, 0)$

b) Find equation of chord:

$x_1 = 2$

$y - y_1 = m(x - x_1)$

$y_1 = 0$

$y = 2(x - 2)$

$m = 2$

$\rightarrow y = 2x - 4$

Use to find x :

$\frac{x^2}{4} - \frac{(2x-4)^2}{6} = 1$

$\times 12$

$3x^2 - 2(2x-4)^2 = 12$

$3x^2 - 2[4x^2 - 16x + 16] = 12$

$3x^2 - 8x^2 + 32x - 32 = 12$

$0 = 5x^2 - 32x + 44$

$(5x - 22)(x - 2) = 0$

\swarrow

\searrow

$x = 2 \quad (P)$

$5x - 22 = 0$

$\rightarrow x = 22/5 \quad (Q)$

$y = 2x - 4 \rightarrow 2(22/5) - 4 = 24/5$

$\therefore Q = (4.4, 4.8)$

9) a) i) As $x \rightarrow \infty$, $f(x) \rightarrow 1 \therefore y=1$ is asymptote

ii) Denominator needs to $\neq 0$

$x^2 + 9 = 0$ has no real solutions

b) Let $f(x) = k \rightarrow k = \frac{x^2 + 4x}{x^2 + 9}$

$\rightarrow k(x^2 + 9) = x^2 + 4x$

$\rightarrow kx^2 + 9k = x^2 + 4x$

$\rightarrow kx^2 - x^2 - 4x + 9k = 0$

$\rightarrow (k-1)x^2 - 4x + 9k = 0$

For equal roots, $b^2 - 4ac = 0$

$$\rightarrow (-4)^2 - 4 \times (k-1) \times 9k = 0$$

$$\rightarrow 16 - 36k(k-1) = 0$$

$$\rightarrow 16 - 36k^2 + 36k = 0$$

$$\rightarrow 36k^2 - 36k - 16 = 0$$

$$\rightarrow 9k^2 - 9k - 4 = 0$$

c) Solve: $9k^2 - 9k - 4 = 0$

$$\rightarrow (3k+1)(3k-4) = 0$$

$$3k+1=0$$

$$\rightarrow k = -1/3$$

$$\rightarrow y = -1/3$$

Find x

$$(k+1)x^2 - 4x + 9k = 0$$

$$-4/3 x^2 - 4x + 3 = 0$$

$$-4x^2 - 12x + 9 = 0$$

$$4x^2 + 12x + 9 = 0$$

$$(2x+3)(2x+3) = 0$$



$$2x+3=0$$

$$\rightarrow x = -3/2$$

Co-ordinate of SP = $(-3/2, -1/3)$

$$3k-4=0$$

$$\rightarrow k = 4/3$$

$$\rightarrow y = 4/3$$

Find x

$$(k+1)x^2 - 4x + 9k = 0$$

$$1/3 x^2 - 4x + 12 = 0$$

$$x^2 - 12x + 36 = 0$$

$$(x-6)(x-6) = 0$$

$$\downarrow x = 6$$

Co-ordinates of SP = $(6, 4/3)$